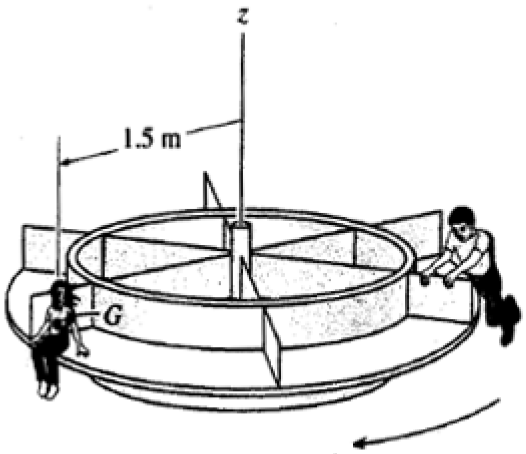
DİNAMİK SINAVLARA HAZIRLIK, ÇALIŞMA SORULARI

DAİRESEL HAREKETTE KUVVET VE İVME

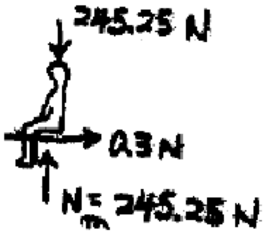
Örnek

A girl having a mass of 25 kg sits at the edge of the merry-go-round so her center of mass G is at a distance of 1.5 m from the axis of rotation. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which she can have before she begins to slip off the merry-go-round. The coefficient of static friction between the girl and the merry-go-round is $\mu_s = 0.3$.



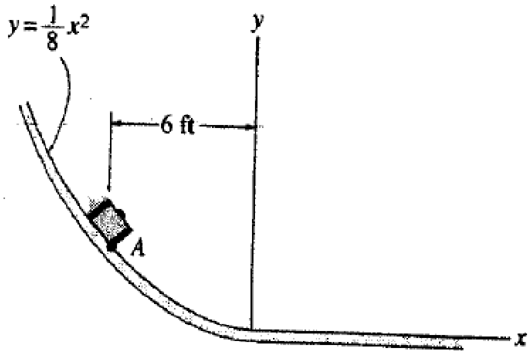
$$\overset{+}{\rightarrow}\Sigma F_n = m a_n; \quad 0.3(245.25) = 25\left(\frac{v^2}{1.5}\right)$$

$$v = 2.10 \text{ m/s} \qquad \textbf{Ans}$$



Örnek

The 10-lb suitcase slides down the curved ramp for which the coefficient of kinetic friction is $\mu_k = 0.2$. If at the instant it reaches point A it has a speed of 5 ft/s, determine the normal force on the suitcase and the rate of increase of its speed.



$$y = \frac{1}{8}x^2$$

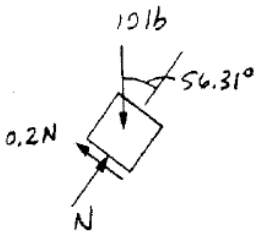
$$\frac{dy}{dx} = \tan\theta = \frac{1}{4}x\Big|_{x=-6} = -1.5 \qquad \theta = -56.31°$$

$$\frac{d^2y}{dx^2} = \frac{1}{4}$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(-1.5)^2\right]^{\frac{3}{2}}}{\left|\frac{1}{4}\right|} = 23.436 \text{ ft}$$

$$+\nearrow\Sigma F_n = ma_n; \qquad N - 10\cos 56.31° = \left(\frac{10}{32.2}\right)\left(\frac{(5)^2}{23.436}\right)$$

$$N = 5.8783 = 5.88 \text{ lb} \qquad \textbf{Ans}$$

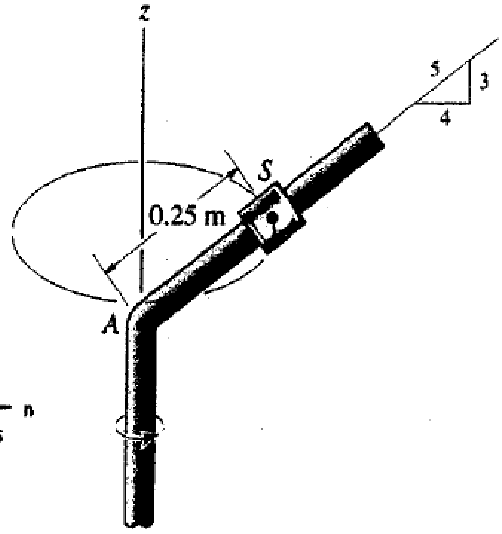


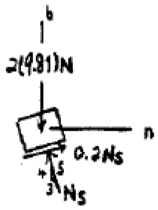
$$+\searrow\Sigma F_t = ma_t; \qquad -0.2(5.8783) + 10\sin 56.31° = \left(\frac{10}{32.2}\right)a_t$$

$$a_t = 23.0 \text{ ft/s}^2 \qquad \textbf{Ans}$$

Örnek

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.





$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$

$$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n; \qquad N_s\left(\frac{3}{5}\right) - 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

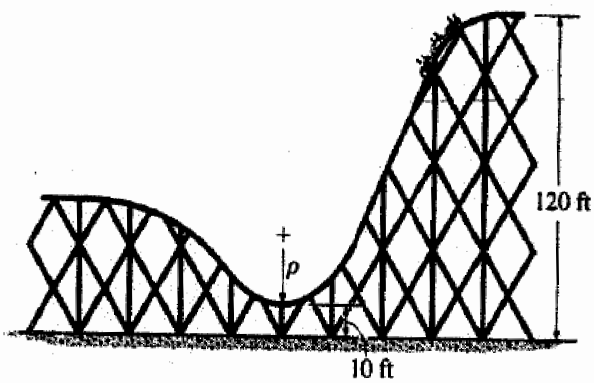
$$+\uparrow \Sigma F_b = m\,a_b; \qquad N_s\left(\frac{4}{5}\right) + 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

$$N_s = 21.3 \text{ N}$$

$$v = 0.969 \text{ m/s} \qquad \textbf{Ans}$$

Örnek

Roller coasters are designed so that riders will not experience more than 3.5 times their weight as a normal force against the seat of the car. Determine the smallest radius of curvature ρ of the track at its lowest point if the car has a speed of 5 ft/s at the crest of the drop. Neglect friction.



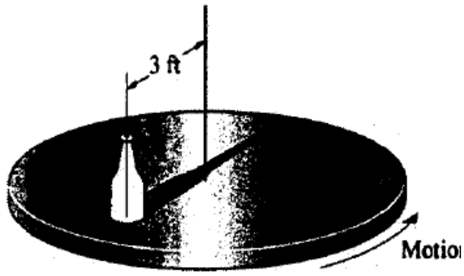
Equation of Motion : It is required that $N = 3.5W$. Applying Eq. 13 – 7, we ave

$$\Sigma F_n = ma_n; \qquad 3.5W - W = \left(\frac{W}{32.2}\right)\left(\frac{7109}{\rho}\right)$$

$$\rho = 88.3 \text{ ft} \qquad \textbf{Ans}$$

Örnek

Work Prob. R1–51 assuming that the platform starts rotating from rest so that the speed of the bottle is increased at 2 ft/s^2.

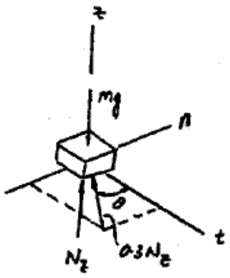


$\Sigma F_z = m a_z;\quad N_z - mg = 0 \quad N_z = mg$

$\Sigma F_t = m a_t;\quad -0.3(mg)\cos\theta = -m(2) \quad \theta = 78.05°$

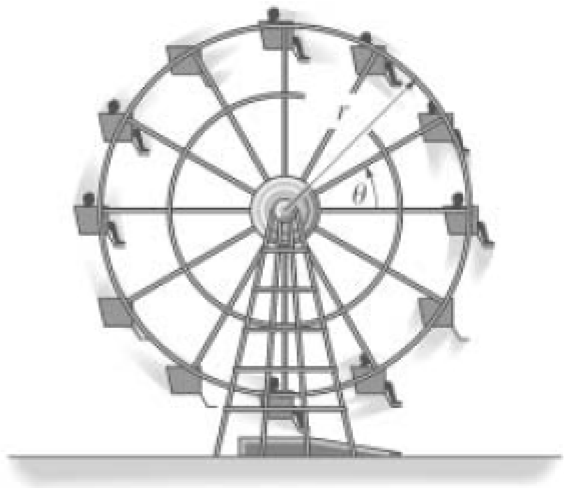
$\Sigma F_n = m a_n;\quad 0.3(mg)\sin 78.05° = m\left(\frac{v^2}{3}\right)$

$v = 5.32$ ft/s Ans



Örnek

The Ferris wheel turns such that the speed of the passengers is increased by $a_t = bt$. If the wheel starts from rest when $\theta = 0°$, determine the magnitudes of the velocity and acceleration of the passengers when the wheel turns $\theta = \theta_1$.



Given:

$b = 4\ \frac{\text{ft}}{\text{s}^3} \qquad \theta_1 = 30\ \text{deg} \qquad r = 40\ \text{ft}$

Solution:

Guesses $\quad t_1 = 1\ \text{s} \qquad v_1 = 1\ \frac{\text{ft}}{\text{s}}$

$a_{t1} = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $\quad a_{t1} = b\,t_1 \qquad v_1 = \left(\frac{b}{2}\right)t_1^{\,2} \qquad r\theta_1 = \left(\frac{b}{6}\right)t_1^{\,3}$

$$\begin{pmatrix} a_{t1} \\ v_1 \\ t_1 \end{pmatrix} = \text{Find}(a_{t1}, v_1, t_1) \qquad t_1 = 3.16\ \text{s} \qquad v_1 = 19.91\ \frac{\text{ft}}{\text{s}} \qquad a_{t1} = 12.62\ \frac{\text{ft}}{\text{s}^2}$$

$$a_1 = \sqrt{a_{t1}^{\,2} + \left(\frac{v_1^{\,2}}{r}\right)^2} \qquad v_1 = 19.91\ \frac{\text{ft}}{\text{s}} \qquad a_1 = 16.05\ \frac{\text{ft}}{\text{s}^2}$$

Örnek

At a given instant the train engine at E has speed v and acceleration a acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.

The truck travels at speed v_0 along a circular road that has radius ρ. For a short distance from $s = 0$, its speed is then increased by $dv/dt = bs$. Determine its speed and the magnitude of its acceleration when it has moved a distance s_1.

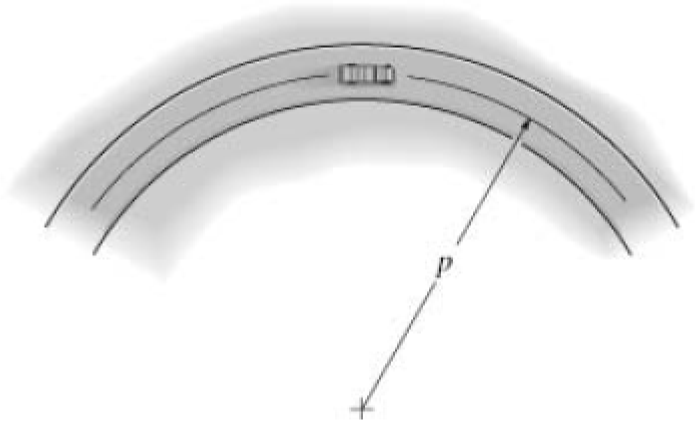
Given:

$$v_0 = 4\ \frac{\text{m}}{\text{s}}$$

$$\rho = 50\ \text{m}$$

$$b = \frac{0.05}{\text{s}^2}$$

$$s_1 = 10\ \text{m}$$



Solution:

$$a_t = v\left(\frac{d}{ds}v\right) = bs \qquad \int_{v_0}^{v_1} v\,dv = \int_0^{s_1} bs\,ds \qquad \frac{v_1^2}{2} - \frac{v_0^2}{2} = \frac{b}{2}s_1^2$$

$$v_1 = \sqrt{v_0^2 + bs_1^2} \qquad v_1 = 4.58\ \frac{\text{m}}{\text{s}}$$

$$a_t = bs_1 \qquad a_n = \frac{v_1^2}{\rho} \qquad a = \sqrt{a_t^2 + a_n^2} \qquad a = 0.653\ \frac{\text{m}}{\text{s}^2}$$

Örnek

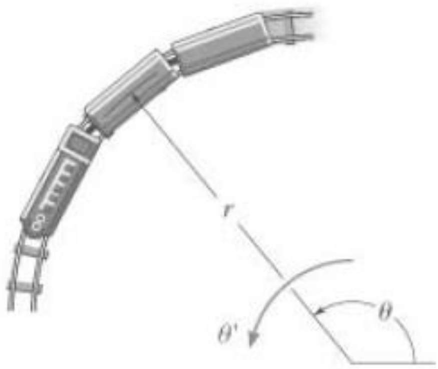
A train is traveling along the circular curve of radius r. At the instant shown, its angular rate of rotation is θ', which is decreasing at θ''. Determine the magnitudes of the train's velocity and acceleration at this instant.

Given:

$$r = 600\ \text{ft}$$

$$\theta' = 0.02\ \frac{\text{rad}}{\text{s}}$$

$$\theta'' = -0.001\ \frac{\text{rad}}{\text{s}^2}$$

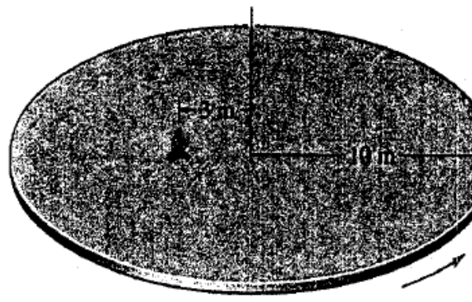


Solution:

$$v = r\theta' \qquad v = 12\ \frac{\text{ft}}{\text{s}}$$

$$a = \sqrt{\left(-r\,\theta'^2\right)^2 + \left(r\theta''\right)^2} \qquad a = 0.646\ \frac{\text{ft}}{\text{s}^2}$$

13-78. The man has a mass of 80 kg and sits 3 m from the center of the rotating platform. Due to the rotation his speed is increased from rest by $\dot{v} = 0.4\ \text{m/s}^2$. If the coefficient of static friction between his clothes and the platform is $\mu_s = 0.3$, determine the time required to cause him to slip.



$\Sigma F_t = m\,a_t;\quad F_t = 80(0.4)$

$F_t = 32\ \text{N}$

$\Sigma F_n = m\,a_n;\quad F_n = (80)\dfrac{v^2}{3}$

$F = \mu_s N_m = \sqrt{(F_t)^2 + (F_n)^2}$

$0.3(80)(9.81) = \sqrt{(32)^2 + ((80)\frac{v^2}{3})^2}$

$55\,432 = 1024 + (6400)(\frac{v^4}{9})$

$v = 2.9575\ \text{m/s}$

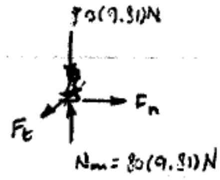
$a_t = \dfrac{dv}{dt} = 0.4$

$\int_0^v dv = \int_0^t 0.4\,dt$

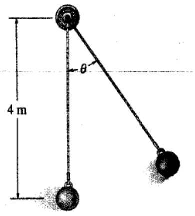
$v = 0.4\,t$

$2.9575 = 0.4\,t$

$t = 7.39\ \text{s}$ **Ans**



13-69. The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



$+\nwarrow \Sigma F_n = ma_n;\quad T - 30(9.81)\cos\theta = 30\left(\dfrac{v^2}{4}\right)$

$+\nearrow \Sigma F_t = ma_t;\quad -30(9.81)\sin\theta = 30a_t$

$a_t = -9.81\sin\theta$

$a_t\,ds = v\,dv$ Since $ds = 4\,d\theta$, then

$-9.81\int_0^\theta \sin\theta\,(4\,d\theta) = \int_4^v v\,dv$

$9.81(4)\cos\theta\Big|_0^\theta = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$

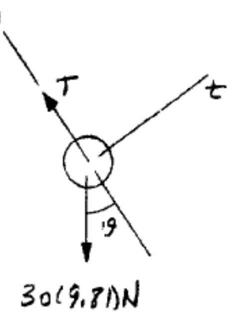
$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$

At $\theta = 20°$

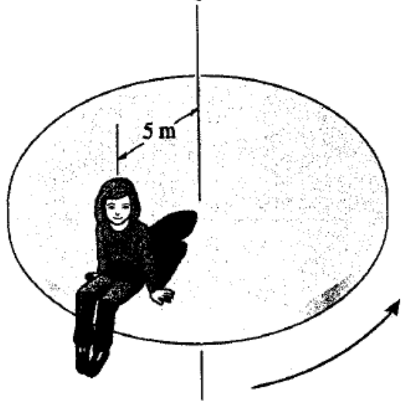
$v = 3.357\ \text{m/s}$

$a_t = -3.36\ \text{m/s}^2 = 3.36\ \text{m/s}^2 \swarrow$ **Ans**

$T = 361\ \text{N}$ **Ans**



13-55. A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.

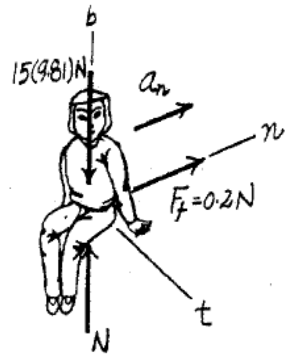


Equation of Motion : Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13 – 8, we have

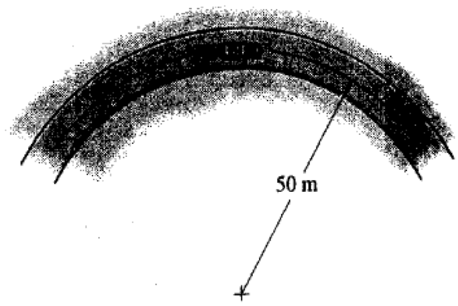
$$\Sigma F_b = 0; \qquad N - 15(9.81) = 0 \qquad N = 147.15 \text{ N}$$

$$\Sigma F_n = ma_n; \qquad 0.2(147.15) = 15\left(\frac{v^2}{5}\right)$$

$$v = 3.13 \text{ m/s} \qquad \textbf{Ans}$$



12-133. The truck travels at a speed of 4 m/s along a circular road that has a radius of 50 m. For a short distance from $s = 0$, its speed is then increased by $\dot{v} = (0.05s)$ m/s^2, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



Velocity : The speed v in terms of position s can be obtained by applying $vdv = ads$.

$$vdv = ads$$

$$\int_{4\text{m/s}}^{v} vdv = \int_0^s 0.05s\,ds$$

$$v = \left(\sqrt{0.05s^2 + 16}\right) \text{ m/s}$$

At $s = 10$ m, $\quad v = \sqrt{0.05(10^2) + 16} = 4.583 \text{ m/s} = 4.58 \text{ m/s}$ **Ans**

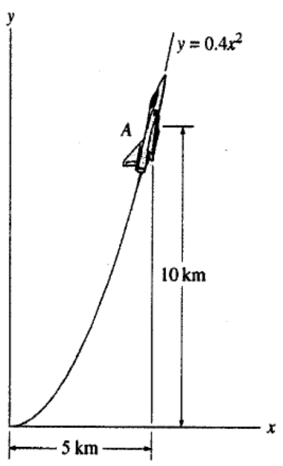
Acceleration : The tangential acceleration of the truck at $s = 10$ m is $a_t = 0.05(10) = 0.500 \text{ m/s}^2$. To determine the normal acceleration, apply Eq. 12 – 20.

$$a_n = \frac{v^2}{\rho} = \frac{4.583^2}{50} = 0.420 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.500^2 + 0.420^2} = 0.653 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-106. The jet plane travels along the vertical parabolic path. When it is at point A it has a speed of 200 m/s, which is increasing at the rate of 0.8 m/s^2. Determine the magnitude of acceleration of the plane when it is at point A.



$$y = 0.4x^2$$

$$\frac{dy}{dx} = 0.8x\Big|_{x=5\text{ km}} = 4$$

$$\frac{d^2y}{dx^2} = 0.8$$

$$\rho = \frac{[1 + (4)^2]^{3/2}}{0.8} = 87.62 \text{ km}$$

$$a_t = 0.8 \text{ m/s}^2$$

$$a_n = \frac{(0.200)^2}{87.62} = 0.457(10^{-3}) \text{ km/s}^2$$

$$a_n = 0.457 \text{ m/s}^2$$

$$a = \sqrt{(0.8)^2 + (0.457)^2} = 0.921 \text{ m/s}^2 \qquad \textbf{Ans}$$